Trajectory Modelling in Shared Spaces: Expert-Based vs. Deep Learning Approach? *

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Abstract. Realistically modelling behaviour and interaction of heterogeneous road users (pedestrians and vehicles) in mixed-traffic zones (a.k.a. shared spaces) is challenging. The dynamic nature of the environment, heterogeneity of transport modes, and the absence of classical traffic rules make realistic microscopic traffic simulation hard problems. Existing multi-agent-based simulations of shared spaces largely use an expert-based approach, combining a symbolic (e.g. rule-based) modelling and reasoning paradigm (e.g. using BDI representations of beliefs and plans) with the hand-crafted encoding of the actual decision logic. More recently, deep learning (DL) models are largely used to derive and predict trajectories based on e.g. video data. In-depth studies comparing these two kinds of approaches are missing. In this work, we propose an expert-based model called *GSFM* that combines Social Force Model and Game theory and a DL model called *LSTM-DBSCAN* that manipulates Long Short-Term Memories and density-based clustering for multi-agent trajectory prediction. We create a common framework to run these two models in parallel to guarantee a fair comparison. Real-world mixed traffic data from shared spaces of different layout are used to calibrate/train and evaluate the models. The empirical results imply that both models can generate realistic predictions, but they differ in the way of handling collisions and mimicking heterogeneous behaviour. Via a thorough study, we draw the conclusion of their respective strengths and weaknesses.

Keywords: Mixed-traffic interaction · Deep learning · Game theory

1 Introduction

In comparison to conventional traffic design where road resources are allocated to road users (agents) by time or space segregation, *shared space* largely removes

* Supported by the German Research Foundation (DFG) through the Research Training Group SocialCars (GRK 1931). The authors thank the participants of the DFG research project MODIS (DFG project #248905318) for providing data sets.

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road signs, signals, and markings, forcing direct interaction between mixed traffic participants (e.g. cars, bikes, pedestrians), guided by informal social protocols and negotiation. This concept was first introduced by Monderman in the 1970s [9]. Shared spaces nowadays can be found in urban areas of many European cities; examples are the Laweiplein intersection in the Dutch town Drachten, Skvallertorget in Norrköping, and Kensington High Street in London [14].

The absence of explicit traffic rules and thereby caused vagueness make it critical to investigate safety issues, especially regarding vulnerable road users (i.e. pedestrians) and traffic efficiency of shared spaces [14]. The foreseeable advent of autonomous driving also raises the need for automated safety systems based on the intent recognition of other road users [11]. However, understanding how road users behave and predicting their actions is far from trivial as these actions are a result of complex decision-making processes from heterogeneous road users.

There is a considerable body of research on microscopic models aimed at tackling these challenges. In particular, we can distinguish two classes of methodologies: the so-called *expert-based* approaches [23, 15, 4, 26, 32, 29, 18] and *data-driven* approaches [2, 21, 17, 27, 13, 7, 8]. Expert-based approaches involve human designers to craft explicit decision rules and corresponding reasoning mechanism to tackle the modelling problem. For example, in the Social Force Model (SFM) [15], the rules of physical dynamics are used to mimic pedestrian movement behaviour in crowded space. Game theory has been used in interaction modelling e.g., users negotiating the right-of-way [29, 18, 5]. However, the requirement of human intervention makes it difficult to scale these models for large or new problems. On the other hand, *data-driven modelling approaches* can be trained by processing the data extracted from real-world situations and deriving a complex neural network structure with associated parameters or weights optimised via training [20]. Examples are e.g. Social-LSTM [2] and Social-GAN [13]. These models are often black boxes, making them hard to understand and explain for humans; The human modeller's intention to guide the models to capture specific desired patterns is difficult to support [16]. Up to now, there is no easy way to interpret the latent features used by a DL model, especially when the structure is of very high dimensionality. Thus, a lack of reliable control of the model may lead to faulty or counter-intuitive behaviour. Besides, computational cost can be a bottleneck for DL-based models [28].

However, it is not easy to fairly compare the expert-based and DL approaches in modelling and predicting mixed traffic trajectories. Firstly, it is difficult to create a common framework that both models can share for a fair comparison. Moreover, they may have different criteria in terms of performance. As an example, expert approaches focus on generating realistic trajectories for agents in simulation, while data-driven approaches focus on predicting trajectories as close as possible to the real trajectories, the so-called ground truth. Hence, the input and output of these approaches are often different.

To our knowledge, there are no studies that compare expert-based and DL approaches for microscopically modelling complex socio-technical systems, namely, shared spaces. Our contributions are summarised below:

- We pursue two models: an expert-based (GSFM, combining a game-theoretic and physics-based model) and a DL model (LSTM-DBSCAN, Long Short-Term Memories with Density-Based Spatial Clustering of Applications with Noise, [10]) .
- We create a common framework for a fair comparison. These two models take the same data as input and generate predictions in the same format.
- The accuracy (in terms of realistic behaviour) of these two models is tested on real-world shared-space scenarios using the same evaluation metrics. Their strengths and weaknesses are experimentally compared and analysed.

2 Methodology

2.1 Problem Formulation

The prediction task is to generate realistic and collision-free future trajectories of the vehicle and pedestrian agents in shared spaces. As preparation for empirical data, all the trajectories with discrete time steps of 0.5 seconds e.g. $(x_i^t, y_i^t) \in \mathbb{R}^2$ on a 2D plane are received from video sequences recorded by static cameras, where x and y are pixel coordinates for the given video, which can be easily converted to meters using the given scale, i stands for agent ID and t for time step. The time steps in observation are $\{1, \cdots k\}$ and the time steps in prediction are $\{k+1, \cdots, m\}$. Accordingly, the visible trajectories for N agents are denoted as $\mathbf{X} = X_1, X_2, ..., X_n$, where $X_i = \sum_{t=1}^{k}(x_i^t, y_i^t)$ and $i \in N$. The prediction of the future trajectories are $\hat{\mathbf{Y}} = \hat{Y_1}, \hat{Y_2}, ..., \hat{Y_n}$, respectively. The task is to predict each agent's location at prediction time steps based on the locations at observation time steps for both DL and expert-based models. Thus, the objective is to minimise $L(\mathbf{Y}, \hat{\mathbf{Y}})$, where $\hat{\mathbf{Y}} = f(\mathbf{X})$ and $\mathbf{Y}$ is the ground truth, $f(.)$ stands for the prediction models, and $L(.,.)$ the loss function.

2.2 Game-Theoretic Social Force Model

We pursue an expert-based approach, called Game-Theoretic Social Force Model (GSFM) [18]. In GSFM, the movement of each agent is modelled in three modules: *trajectory planning*, *force-based modelling*, and *game-theoretic decision-making*. Each module has different roles to perform. GSFM build on a BDI (**B**elief, **D**esire, **I**ntention) platform, LightJason [3], to design and explain the control flow among the modules. The BDI controller acts as the brain of the agent to perceive the environment and activate one of these modules based on the situation. Each module triggers the controller on the completion of their respective task(s). Fig. 1 visualises the overall structure of the GSFM model.

The trajectory planning module computes free-flow trajectories for each agent by considering static obstacles like boundaries, or trees in the shared space.

The force-based modelling and game-theoretic decision modules are responsible for modelling interactions among agents. In GSFM, these interactions are classified into two categories based on the observation of the video data and on the classification of road users' behaviour given by Helbing et al.[15]: simple interaction (percept → act) and complex interaction (percept → choose an action among many alternatives → act).

The force-based module handles simple interactions. It uses the classical SFM to capture the driving force of each agent towards their destination ($\boldsymbol{D}_i^o$), the repulsive force from static obstacle ($\boldsymbol{I}_{iW}$) and from other pedestrian ($\boldsymbol{I}_{ij}$), and extends SFM to capture car following interaction ($\boldsymbol{I}_{\text{following}}$) and pedestrian-to-vehicle reactive interaction ($\boldsymbol{I}_{\text{stopping}}$). $\boldsymbol{I}_{\text{stopping}}$ happens only if pedestrian(s) have already initiated walking in front to the vehicle, so the vehicle decelerates to let the pedestrian(s) pass.

The game-theoretic module is responsible for handling complex interactions i.e. pedestrian(s)-to-car(s) or car-to-car interaction. A sequential leader-follower game, a.k.a. Stackelberg game is used to handle these interactions. In such a game, both leader and follower players try to maximise their utility: the leader player chooses a strategy first by considering all possible reactions of follower players and the followers react based on the chosen strategy of the leader [29]. The sub-game perfect Nash equilibrium (SPNE) is applied to find the optimal strategy pair, denoted by Eq. (1).

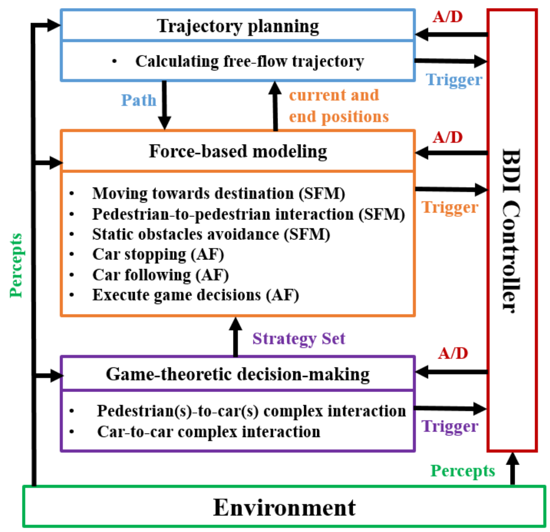


Fig. 1: Trajectory prediction in shared spaces using GSFM. Here, **AF** means added force to classical SFM and **A/D** indicates activation/deactivation of a module in GSFM.

$$\text{SPNE} = \{s_l \in S_l | max(u_l(s_l, Bs_f(s_l)))\}, \forall s_l \in S_l. \tag{1}$$

$$Bs_f(s_l) = \{s_f \in S_f | max(u_f(s_f|s_l))\}. \tag{2}$$

The equation (2) is the best answer from the follower. Here, s_l, s_f, u_l, u_f and S_l, S_f are the leader's and followers' strategies, utilities regarding the respective strategies and their strategy sets respectively. In GSFM, *Continue*, *Decelerate* and *Deviate* (pedestrian only) are the possible strategies for agents. Each complex interaction is resolved by playing an individual Stackelberg game and the games are not dependent on each other. For any game, the number of leaders is set to one and followers to one or more, and the faster agent (i.e. car) is chosen as the leader. If any complex situation involves more than one cars e.g, pedestrian(s)-to-cars interaction, then the one who detects the conflict first is set as leader. The details of these modules e.g. payoff estimation or interaction modelling, and categorisation and recognition of interaction is given in [18].

Although these modules take control alternatively, at the start of the simulation, GSFM maintains a hierarchy among its modules: it starts with the trajectory planning with the assumption that agents plan their trajectory before starting walking/driving physically. Once agent gets there trajectory, force-based module is activated to execute their physical movement. Conflict recognition is performed at regular intervals. Based on the situation context (i.e. simple or complex conflict), the BDI controller activates either force-based or game module to decide on strategies. Once the strategies are decided, force-based module is activated again (if not activated already) to execute them. The BDI controller also prioritises the decision taken by these modules i.e. $\boldsymbol{I}_{\text{game}}$ takes precedence over decision of other modules, except for $\boldsymbol{I}_{\text{stopping}}$, with the premise that complex interaction e.g. car-to-pedestrian is more critical than pedestrian-to-pedestrian or car following interaction.

To sum up, the process of GSFM for predicting the movement behaviour of any target agent i in any time step t is presented in Eq. (3)–(5). Here, i, j, W, Z_i, X_i^t, and $Y^{t+\Delta t}$ depict the target agent, competitive pedestrian, boundary, input to the model, the agent's position in current and next time step respectively. The input profile Z_i is derived from the observation of X_i, which contains start, goal, speed profile of i, and minimum distance acceptance of i with others. The goal of i is estimated by using the heading in the last observed position and average speed over the observed time steps.

$$\text{Pedestrian: } \frac{d\overrightarrow{v^t}_i}{dt} = \left(\overrightarrow{D}_i^o + \Sigma\overrightarrow{I}_{iW} + \Sigma\overrightarrow{I}_{ij}\right) \textit{ or } \overrightarrow{I}_{\text{game}}, \tag{3}$$

$$\text{Car: } \frac{d\overrightarrow{v^t}_i}{dt} = \overrightarrow{D}_i^o \textit{ or } \overrightarrow{I}_{\text{following}} \textit{ or } \overrightarrow{I}_{\text{game}} \textit{ or } \overrightarrow{I}_{\text{stopping}}, \tag{4}$$

$$\hat{Y}_i^{t+\Delta t} = f(Z_i, (\frac{d\overrightarrow{v^t}_i}{dt} + X_i^t)). \tag{5}$$

2.3 LSTM with DBSCAN

We pursue a DL model, called Long Short-Term Memories with Density-Based Spatial Clustering of Applications with Noise (LSTM-DBSCAN). For a target agent i, $f(X_i)$ is LSTM-DBSCAN that takes X_i as input and outputs $\hat{Y}_i$. The LSTM-DBSCAN contains two modules: a *mapping module* for interaction pooling and an *LSTM module* for motion planning, see Fig. 2.

The mapping module is used for pooling the interactions between the target agent and other neighbourhood agents at each time step. It follows the idea of repulsive force in SFM [15] to map the collision probability based on safety distance maintained by the target and neighbourhood agents, denoted by probability density mapping (PDM). Safety distance d (see Fig. 2) is measured from the approximate mass points from the target agent to the neighbourhood agent. If two agents approach each other, PDM increases exponentially. In addition, we follow the same idea as [8] to extend safety distance with buffers for pedestrian personal space [12] and car geometry, denoted by the egg shapes with approximate radius (r or R) in Fig. 2. Radius are extracted from real-world interactions with the differentiation of road users' transport mode.

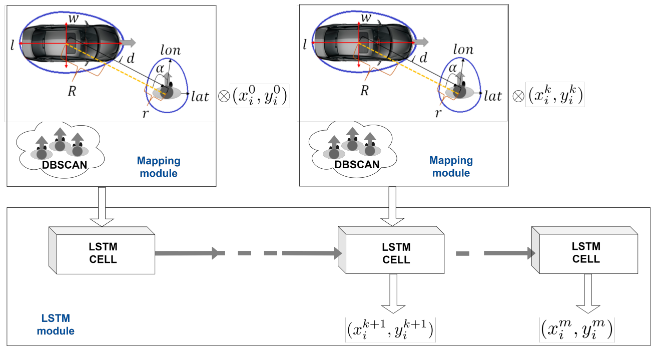


Fig. 2: The structure of LSTM-DBSCAN for target agent i. $\otimes$ stands for the concatenation of the output of the mapping module and the target agent's position at each time step.

However, short distance does not necessarily indicate high collision probability. Pedestrians from one group tend to walk at the same speed and maintain a certain distance, to synchronise their speed and distance for communication and visibility between each other [30, 26]. Therefore, inside the mapping module, a DBSCAN cluster is incorporated to detect pedestrian groups, so as to cancel out erroneous collision indication and relax on close interactions for group members. At each time step in the observation time, present agents are clustered. The minimum number of points (MinPts) is set to two as the smallest group (cluster) only contains two agents. The maximum Euclidean distance (ϵ) from neighbourhood point to the core points in a DBSCAN cluster is set to one meter. A neighbourhood agent is defined as a group member for the target agent if they co-exist in the same cluster over 90 % of the observed time steps. Both ϵ and the overlap ratio of time steps are decided from the hyper-parameter searching in [6]. During clustering, PDM is reset to zero for group members.

The LSTM module is used for motion planning, which takes the target agent's coordinates and the interactions with neighbourhood agents using PDM as input at each observed time step. In prediction time, similar to Social-LSTM [2], the LSTM module uses the encoded information from observed time steps to predict the distribution of the next positions. While, our DL model differs from Social-LSTM by semantically quantifying all the neighbourhood agents' impact using a collision probability, instead of occupancy grid within a predefined interactive zone using binary values. It also differentiates the impact of group members and non-group members on the target agent from a DBSCAN cluster.

In short, Eq. (6) describes the prediction process for the target agent i. For simplicity, the time step is omitted in the equation. $f(.,.)$ stands for LSTM, $\phi(.)$ for PDM, and $\psi(.,.)$ for DBSCAN.

$$\hat{Y}_{i\in N} = f(X_{i\in N}, \phi(\psi(X_{i\in N}, X_{j\in N, j\neq i}))) \tag{6}$$

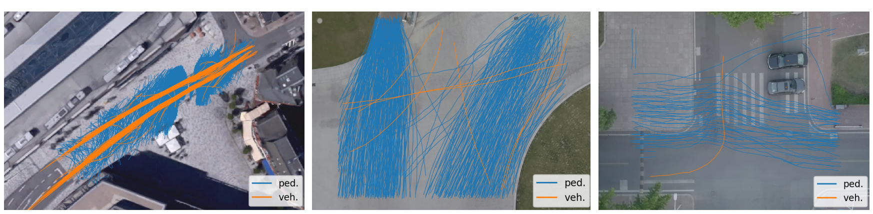
3 Data Sets and Evaluation Metrics

3.1 Data Sets

To evaluate the performance of the proposed two models, we use two data sets with mixed traffic trajectories extracted from shared spaces of different layout, namely, the Hamburg Bergedorf station data set (HBS) from Germany [24] and the DUT data set from the campus of Dalian University of Technology in China [31]. The layout of HBS is a street with pedestrian crossing from both sides. The DUT data set has 11 clips recorded in a roundabout and 17 clips recorded in an intersection. The clip from HBS contains dynamic pedestrians-to-vehicles interactions. Whereas, the clips from DUT have less vehicles, but more vehicles-to-crowd interactions [31]. Table 1 summarises the statistics for each data set. The first 1200 time steps of the HBS data set and 12 clips (8 from the intersection and 4 from the roundabout) from the DUT data set are used for extracting interaction scenarios for evaluation. In total, we manually extracted 89 scenarios that involve interactions between pedestrians and vehicles: 67 scenarios from HBS and 22 from DUT. Please note that due to the short length of clips from DUT, scenarios extracted from DUT are shorter than the ones from HBS. The rest of the two data sets are used for calibrating the expert-based model GSFM, and training the DL model LSTM-DBSCAN. There is no overlap between the evaluation and training data.

Table 1: Statistics for each data set

Data set	#Time steps	Time-step duration	#Ped	#Veh	Layout description
HBS	3620	0.5 seconds	1115	338	1 clip in a street
DUT	648	0.5 seconds	1767	69	11 clips in a roundabout 17 clips in an intersection



(a) HBS (b) DUT roundbout (c) DUT intersection

Fig. 3: Mixed trajectories from shared spaces of different layout

3.2 Evaluation Metrics

To evaluate the performance of GSFM and LSTM-BDSCAN, we use displacement (Euclidean and Hausdorff distance) and heading errors as metrics. As commonly used in other works [2, 13], the average Euclidean distance error (ADE) measures the aligned error for each step and we report the value averaged over the path. For the accumulated error, we used Hausdorff distance to measure the largest distance from the set of the predicted positions of a trajectory to the set of true positions [22]. In most cases, the displacement error accumulates with the increment of time steps. The Hausdorff distance error is very similar to the displacement error for the final position. Heading (from the previous position to the next position) error measures the pairwise absolute heading difference over all positions between the predicted and ground truth trajectories.

Due to the stochastic characteristics of human movement behaviour, different road users may behave in different ways in a given situation [19]. In this regard, it is very difficult to quantify which way of behaving is better than the other. The quantitative evaluation metrics alone may not be sufficient to demonstrate the feasibility of a trajectory prediction model. Therefore, we perform case studies to analyse how both proposed models handle different real-world scenarios.

4 Experimental Results

GSFM is implemented using a BDI multi-agent framework, LightJason [3]. LSTM-DBSCAN is implemented using tensorflow [1] framework. The LSTM units have a size of 128 and one vertical layer. It is trained using RMSProp optimised with a learning rate of 0.003 and batch size of 16 for 300 epochs. The observation sequence length is set to six time steps and the prediction sequence length varies with a minimum length of six time steps. Both GSFM and LSTM-DBSCAN are tested on real-world scenarios lasting different length of time steps, unlike [2, 13, 27] that predict trajectories of a fixed length of time steps.

4.1 Quantitative Results for Individual Models

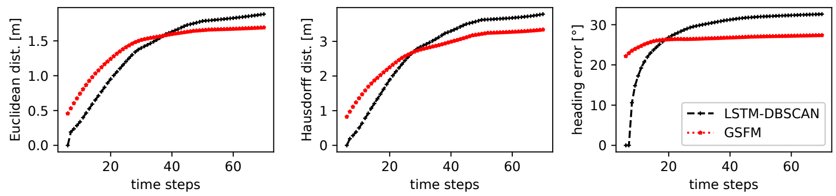
Fig. 4 shows the comparison among the ground truth trajectories and the trajectory predictions by LSTM-DBSCAN and GSFM along time horizon on HBS and DUT, measured by Euclidean and Hausdorff distance, and heading error.

In general, as the time step increases, the performance of both models decreases on both data sets, as the uncertainty increases further into the future. Fig. 4a shows that LSTM-DBSCAN performs better in short-sequence prediction (approximately 25 time-steps) than GSFM by all measurements for the HBS data set, which contains many long-sequence interactions. However, the performance of LSTM-DBSCAN degrades faster than GSFM with the increment of time steps.

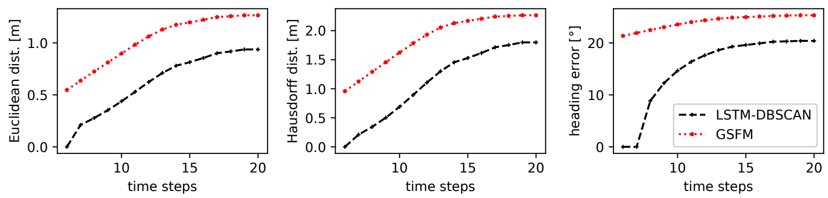
From Fig. 4b, the performance for LSTM-DBSCAN on DUT is significantly better than GSFM regarding all the evaluation metrics. As mentioned before (see Section 3.1), the scenarios from DUT are shorter and more complicated due to the high density of traffic in the intersection and the roundabout than HBS. Both of the proposed models have a limited capacity to deal with dense traffic.

4.2 Qualitative Results for Individual Models

Fig. 5 shows the predictions made by GSFM and LSTM-DBSCAN in different scenarios. In most scenarios denoted in the sub-figures, both GSFM and LSTM-



(a) The performance on HBS data set



(b) The performance on DUT data set

Fig. 4: The performance of GSFM and GSFM-w-LSTM on different data sets.

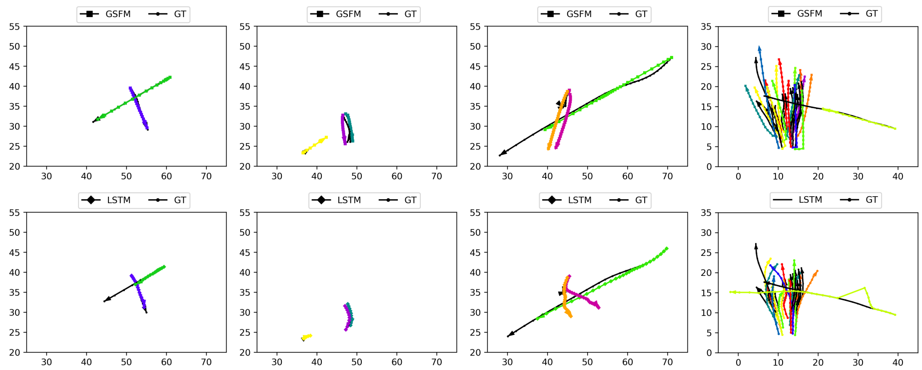
DBSCAN generate feasible trajectories in interactions within a small number of road users from the HBS data set. Whereas, both models have limited performance in dealing with dense traffic in the DUT data set.

Based on the visualisation, the prediction from the GSFM model overlaps the ground truth well and outperforms the LSTM-DBSCAN model when the trajectories have a constant heading direction (see Fig. 5a).

However, when road users change their heading direction, GSFM may have difficulty mimicking this behaviour. As can be seen in Fig. 5b, the trajectories generated by GSFM are straight forward and homogeneous as the model only specifies a limited number of behaviour patterns based on the assumption of relatively fixed speed (i.e. a Gaussian distribution of speed). Else-ways, LSTM-DBSCAN can automatically capture both the speed and orientation attributes of each road user based on a short observation time.

Moreover, GSFM and LSTM-DBSCAN handle conflicts differently. GSFM deals with conflicts explicitly either based on the social forces, where the repulsive force increases exponentially when two road users come closer [15] or by game playing to negotiate on the priority over road spaces. In contrast, LSTM-DBSCAN learns collision avoidance based on the training data with probability density mapping automatically. They may generate different negotiating results even facing the same interactions. For example, in Fig. 5c, both GSFM and LSTM-DBSCAN predict that both pedestrians crossing the street before the upcoming vehicle, although LSTM-DBSCAN predicts a more aggressive behaviour for the vehicle which results in a near collision with the crossing pedestrians.

In Fig 5d, both GSFM and LSTM-DBSCAN do not optimally predict the trajectory for the vehicle approaching a large number of pedestrians. In GSFM, the vehicle decelerates and some of the pedestrians accelerate for collision avoidance. Whereas, LSTM-DBSCAN generates a very unfeasible trajectory for the vehicle, which results in pedestrians deviating from the upcoming vehicle.



(a) HBS scenario 1 (b) HBS scenario 2 (c) HBS scenario 3 (d) DUT scenario 1

Fig. 5: Comparison of the predictions by GSFM and LSTM-DBSCAN. Ground truth trajectories are in black colour and predicted trajectories are colour-coded. Vehicles are travelling in either diagonal or slightly horizontal directions. The arrows indicate the moving directions of pedestrians and vehicles.

4.3 Pros and Cons of GSFM and LSTM-DBSCAN

Based on the empirical results, we summarise the strengths and weaknesses of the GSFM and LSTM-DBSCAN models in Table 2.

Table 2: Pros and cons of GSFM and LSTM-DBSCAN

Model	GSFM	LSTM-DBSCAN
Pros	transparent, explainable, collision-free trajectories, no need for labelled data, easy to control	less domain knowledge, not based on rules, good short-term predictions, realistic predictions in simple scenarios
Cons	domain knowledge, complicated rules, homogeneous predictions, inflexible in scaled problems, limited in dense traffic	not transparent, not explainable, collision-free trajectories not guaranteed, computationally inefficient, might be over-fitted, limited in dense traffic, require labelled data, hard to control

Some pioneer studies [25, 16] indicate that a hybrid model can be used to hoard the collective advantages of both kinds of approaches. Therefore, in future, we consider to combine the expert-based and DL approaches to model collision-free, explainable, and heterogeneous trajectories of agents.

5 Conclusion and Future Work

In this study, we propose an expert-based model and a deep learning model for mixed traffic trajectory modelling and prediction in shared spaces of different layout. Both of the two models take the same input data for a fair comparison. Their performance is evaluated on real-world shared-space scenarios, such as interactions between pedestrians and vehicles. In most cases, both models can predict realistic trajectories for mixed traffic agents. The expert-based model, using Social Force Model and Game theory, predicts collision-free trajectories. While the predictions tend to be homogeneous. The deep learning model that manipulates Long Short-Term Memories and density clustering predicts accurate short-term trajectories. However, its performance decreases significantly for longer-term prediction and it may generate (near) collision predictions. Both models have limited performance in coping with a large number of agents.

To improve the performance and robustness of the individual models, more open-source data sets of shared spaces will be used for training and evaluation. We will build a hybrid model such as by combining the collision-avoidance mechanism of the expert model with the motion planning techniques of the DL model, to predict collision-free and realistic trajectories in mixed traffic environments.

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